Developments of the Three-Dimensional Variational Data
Assimilation System for the Nonhydrostatic GRAPES*

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(Received May 31, 2009)

ABSTRACT

Based on the original GRAPES (Global/Regional Assimilation and PrEdiction System) 3DVAR (p3DVAR), which is defined on isobaric surface, a new three-dimensional variational data assimilation system (m3DVAR) is constructed and used exclusively with the nonhydrostatic GRAPES model in order to reduce the errors caused by spatial interpolation and variable transformation, and to improve the quality of the initial value for operational weather forecasts. Analytical variables of the m3DVAR are fully consistent with predictands of the GRADES model in terms of spatial staggering and physical definition. A different vertical coordinate and the nonhydrostatic condition are taken into account, and a new scheme for solving the dynamical constraint equations is designed for the m3DVAR. To deal with the difficulties in solving the nonlinear balance equation at σ levels, dynamical balance constraints between mass and wind fields are reformulated, and an effective mathematical scheme is implemented under the terrain-following coordinate. Meanwhile, new observation operators are developed for routine observational data, and the background error covariance is also obtained. Currently, the m3DVAR system can assimilate all routine observational data.

Multi-variable idealized experiments with single point observations are performed to validate the m3DVAR system. The results show that the system can describe correctly the multi-variable analysis and the relationship of the physical constraints. The difference of innovation and the analysis residual for \( \mathcal{T} \) also show that the analysis error of the m3DVAR is smaller than that of the p3DVAR. The \( \mathcal{T} \) scores of precipitation forecasts in August 2006 indicate that the m3DVAR system provides reduced errors in the model initial value than the p3DVAR system. Therefore, the m3DVAR system can improve the analysis quality and initial value for numerical weather predictions.

Key words: GRAPES, nonhydrostatic model, data assimilation, numerical prediction

Citation: Ma Xulin, Zhuang Zhaorong, Xue Jishan, et al., 2009: Developments of the three-dimensional variational data assimilation system for the nonhydrostatic GRAPES. Acta Meteor. Sinica, 23(6), 725–737.

1. Introduction

Numerical weather prediction (NWP) is a typical initial value problem in the field of mathematics and physics. High quality initial values are often obtained through assimilation of meteorological observations (Xue, 2004). Therefore, data assimilation is an essential part of a NWP system. It can synthesize all kinds of observational data and transform them into initial value for the NWP model. It also provides an integrated and complete dataset of the real atmosphere, which can be conveniently used. Thus, data assimilation becomes fundamental for the diagnosis and analysis of weather and climate. In essence, the purpose of data assimilation is to obtain an approximation much closer to the real atmosphere based on the model background while syncretizing scientifically observational data following the atmospheric evolution in NWP models and absorbing all other possible observations.

The relationship between the data assimilation system and the numerical model falls usually into two
types: (1) the data assimilation scheme is independent of the NWP, and can be applied conveniently to other forecast models; (2) the data assimilation scheme is specifically designed, developed, and optimized for a forecast model. The advantages of the former are obvious, so some of the major numerical prediction systems adopt this scheme, such as MM5 (mesoscale model version 5) (Barker et al., 2004), WRF (weather research and forecasting) (Skamarock et al., 2005), etc. However, extra transformation of variables and coordinates are needed when the analysis fields are applied to the initial values of the forecast model. This may cause additional errors because of the inconsistence between the data assimilation scheme and the physical constraints of the forecast model. Therefore, many operational NWP systems, such as those of NCEP (National Centers for Environmental Prediction; Parrish and Derber, 1992), ECMWF (European Center for Medium-Range Weather Forecasts; Courtier et al., 1998), France, and Britain (Lorenc et al., 2000), etc., take the latter in practice.

In China, the three-dimensional variational (3DVAR) assimilation system is in general synchronously developed with its forecast model GRAPES (Global/Regional Assimilation and PrE-diction System). The first method was adopted at the beginning due to the absence of the complete numerical prediction model. Most 3DVAR versions of GRAPES were implemented on isobaric surface, i.e., the isobaric surface analysis, or p3DVAR, was carried out. With the significant development and operational application of the GRAPES numerical analysis and forecast system, eliminating extra errors to improve the precision of initial value becomes more imperative. Besides, the development of a four-dimensional variational (4DVAR) data assimilation system also requires the decrease of superfluous processes between model and analysis. Thus, a new 3DVAR system fully compatible with the GRAPES numerical forecast model is developed based on the p3DVAR, and is called briefly as m3DVAR. This new system can directly obtain model forecast variables under the same vertical coordinate of the numerical forecast model.

The data assimilation scheme developed for a certain NWP system is mainly featured by consistent state and forecast variables of the numerical model in terms of both physical property and spatial grid discretization algorithm. Therefore, in the m3DVAR, transforms from state variables to optimal control variables and from state variables to observational variables (commonly known as observation operator), as well as constructions of background error structure and balance constraints among different physical variables all need to undergo corresponding changes. For the hydrostatic model, the main problems come from complicated transforms caused by the terrain-following coordinate. While for the nonhydrostatic model, new problems arise from the incoherence between the forecast variables of the model and the basic observational elements. Even for the routine sounding observation, the associated observation operator is never a simple spatial interpolation transform. Since GRAPES is a nonhydrostatic model, establishment of the 3DVAR system for GRAPES is more complex than a general hydrostatic model.

The main difference between the m3DVAR and the p3DVAR of GRAPES lies in dynamical constraints, background error structure, observation operator, etc. The transform of vertical coordinate and adjustment of horizontal staggering grids are relatively easy. Therefore, this paper is mainly devoted to depicting the design and the primarily different parts of the m3DVAR (in comparison with the p3DVAR) for the GRAPES nonhydrostatic model, and to presenting relevant experiment results. In Section 2, brief descriptions of the GRAPES forecast model and the m3DVAR system are provided. The specific schemes of the m3DVAR are described in detail in Sections 3 and 4. The background error structure of analysis variables is illustrated in Section 5, while the

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2 Chen Dehui, Xue Jishan, Shen Xueshun, et al., 2003: The theoretical design and experiments of the new generation hydrostatic/nonhydrostatic multiscale community numerical prediction model. China meteorological numerical prediction system innovation research technical manual. (internal document)
idealized and real data experiments are introduced in Section 6. The summary and conclusions are given in Section 7.

2. The nonhydrostatic GRAPES and the m3DVAR

GRAPES is a nonhydrostatic numerical forecast model with longitude-latitude grids and a terrain-following vertical coordinate (Gal-Chen and Somerville, 1975). For the convenience of dealing with terrain, the real terrain height \( Z \) is transformed to terrain-following normalized height, i.e., \( \tilde{z} = Z_T/(Z - Z_S) \), where \( Z_S \) and \( Z_T \) is terrain height at surface and top of the model atmosphere, respectively. The discretization scheme of model forecast employs the Arakawa-C staggering scheme (Arakawa and Lamb, 1977) in the horizontal direction, and the Charney-Phillips staggering scheme (Charney and Phillips, 1953) in the vertical. The forecast variables include nondimensional pressure (Exner function) \( \pi = (p/p_0)^{R/C_p} \) \( (p_0 = 1000 \text{ hPa}) \); potential temperature \( \theta \); wind components \( u, v, \) and \( w \); and specific humidity \( q \), respectively. Here, we use \( \tilde{w} \) to represent vertical velocity in the terrain-following coordinate. Thus, the spherical vertical velocity transform is:

\[
\tilde{w} = \frac{Z_T}{Z_T - Z_S}(w - \frac{Z_T - Z}{Z_T - Z_S} \left( \frac{u}{a \cos \varphi} \partial \lambda + \frac{v}{a} \partial \varphi \right)),
\]

where \( w \) denotes the vertical velocity under isobaric surface coordinate, and \( a \) is the earth radius. The variables \( \varphi \) and \( \lambda \) are the latitude and longitude under the spherical coordinate, respectively. The GRAPES model further introduces a static reference atmosphere in hydrostatic balance, with temperature and pressure horizontally uniformly distributed. Thus, the state variables of the GRAPES model are deviations from this reference atmosphere.

The state vectors in the m3DVAR system are completely the same as the model predictands, and they are defined on same vertical levels and horizontal grids as the numerical model, as shown in Figs. 1 and 2. In general, since the state variables are expressed as deviations from background in the variational data assimilation scheme, the analysis increment and its deviation increment are equal. The deviations relative to the reference atmosphere in the forecast model are thus not used in the analysis scheme because the local linearization of the equation in the variational data assimilation scheme cannot use deviations. Since model variables are relatively independent of each other, in order to simplify the complexity of the problem, we follow the similar idea as in p3DVAR, i.e., take physical and dynamical balance transforms among different variables, and adopt independent variables such as cyclostrophic wind, imbalanced pressure, imbalanced divergent wind, and specific humidity as analysis variables (Zhang et al., 2004). The velocity potential \( \chi \) and stream function \( \psi \) are expressed by the wind components \( u \) and \( v \), namely physical transform, and the following balance operators \( M, N, P, \) and \( Q \) are introduced to decompose state variables into balance parts (marked by subscript “\( \text{b} \)” and imbalance parts (marked by subscript “\( \text{u} \)”), i.e.,

\[
\psi = \psi_b, \quad \chi = \chi_b + \chi_u = M(\psi) + \chi_u, \quad \pi = \pi_b + \pi_u = N(\psi) + \pi_u, \quad \theta = \theta_b + \theta_u = Q(\psi) + \theta_u, \quad w = w_b + w_u = P(\psi) + w_u, \quad q = q_b,
\]

where the symbols are the same as above. Using \( X \) to denote the state variables (\( \psi, \chi, \pi, \theta, w, q, \)) and \( X_c \) to denote independent analysis variables (\( \psi, \chi_u, \pi_u, \theta_u, w_u, q, \)) we could briefly rewrite Eqs. (2)–(7) as \( X = P(X_c) \).

Provided that background errors are irrelevant, the new variables (\( \psi, \chi_u, \pi_u, \theta_u, w_u, q, \)) are then used as optimal control variables of the variational data assimilation system, which can simply solve the problem. Detailed forms of the balance relation will be introduced in the next section. If \( B_u \) is the correlation matrix of background error, it is then a block diagonal matrix and can be decomposed into

\[
B_u = UU^T.
\]

Here, superscript “\( T \)” denotes matrix transposition,
where $\omega = U(X_c - X_{cb})$. $\tilde{H}(w) = H(P(U^{-1}\omega + X_{cb}))$ is the observation operator about $\omega$. The derivation of Eq. (9) is similar to that of p3DVAR, and the related solution to the optimal problem is also basically the same as p3DVAR. The only difference lies in the corresponding physical meaning and numerical value. The following sections are then devoted to discussion of new problems caused by physical property changes of variables. Since humidity $q$ is an independent variable, treatment of $q$ is kept the same as that in p3DVAR except for the coordinate transformation, so it is no longer discussed below.

### 3. Dynamical constraints

Introducing a dynamical balance relation in data assimilation has become a common and effective way to maintain the physical compatibility among analysis variables, as well as to solve the optimal problems. The definition of the dynamical balance relation is, however, different in different data assimilation schemes. In general, the balance relation is established by 1) using statistical results, 2) using certain dynamical balance condition such as in MM5 (Barker et al., 2004) and WRF (Skamarock et al., 2005) models, or 3) a combination of the former two methods, where the function type of the balance relation is obtained by the dynamical balance equation, and then the quantitative relation is achieved by statistical results. At present, both the m3DVAR and p3DVAR adopt the first approach (Zhuang et al., 2006). Although GRAPES is a nonhydrostatic model, the nonhydrostatic balance problem at initial time is not considered here. Thus, only one of the model variables $\pi$ and $\theta$ is independent. Currently, $\pi$ is considered to be the state variable, while $\theta$ is deduced from $\pi$. The balance relation is defined as the nonlinear balance equation between wind and pressure, where balanced pressure fields are deduced from stream function through the balance relation. Considering the balance relation between wind and pressure in terms of the model state variables and its vertical coordinate, we know that the balance equation need to be depicted by model variable $\pi$ under terrain-following height levels. Therefore, transformation from $\tilde{z}$ coordinate to isobaric surface, and physical
transformation between $\pi$ and geopotential height $\phi$ have to be settled beforehand. In this case, expression of the balance equation in the terrain-following coordinate is so complicated because of the terrain calculation that its solution is not easily obtained. Therefore, in order to simplify the calculation and satisfy the precision requirement, we design a new scheme to solve the balance equation: introducing a group of ancillary vertical isobaric surface with higher resolution, and transforming control variable $\psi$ from model levels in the terrain-following coordinate through a high precision mathematical scheme onto a group of horizontal isobaric surfaces with a similar vertical resolution of the model, and solving the balance equation on the isobaric surface, then converting the solutions onto model levels in the analysis space. In this way, the problem of directly solving the balance equation in the terrain-following vertical coordinate could be simplified. The above analysis shows that under the hydrostatic balance condition, adopting pressure as an independent variable in the vertical and geopotential height as the predictands that could reflect pressure distribution, the nonlinear balance equation can be written as

$$\pi = S_\pi \phi_p, \quad (13)$$

$$\phi_p = P_\phi \pi. \quad (14)$$

$P_\psi$ and $S_\psi$ are transform operator (spatial interpolation operator; here it is specified to be a cubic spline interpolation operator) of stream function, where $P_\psi$ is from model levels to isobaric surface levels, and $S_\psi$ is for the opposite direction. $P_\phi$ is another transform operator (including spatial interpolation operator and physical transform operator) from nondimensional pressure $\pi$ on model levels to geopotential height $\phi_p$ on isobaric surface, and $S_\phi$ is for the opposite direction. Firstly, transforming the stream function in altitude coordinate of forecast model through Eq. (11) onto a group of predefined isobaric surface levels, and solving Eq. (10) on isobaric surface to get balanced geopotential height $\phi_p$. Then obtaining nondimensional pressure $\pi$ at model levels by vertical interpolation and physical transformation of altitude and nondimensional pressure, namely Eq. (13). In the process of the cubic spline interpolation, when variables are transformed from model levels to isobaric levels, model levels are data node and isobaric surface levels are the interpolated nodes, of which altitude is the independent variable and pressure is the dependent variable, and vice versa. The transform of pressure and altitude acts as inverses of each other, but stays one-to-one correspondence.

The operator $S_\phi$ in Eq. (13) transforms geopotential height at isobaric levels onto model levels through spatial interpolation, and gets nondimensional pressure by the relation of geopotential height and nondimensional pressure at the model levels. Using the increment analysis method in p3DVAR, the process is described as below:

1. Transform geopotential height analysis increment at isobaric surface to the one at model levels by using the cubic spline interpolation operator:

$$\delta \phi = S_\phi \delta \phi_p, \quad (15)$$

where $S_\psi$ is the cubic spline interpolation operator from isobaric surface to model levels, and symbol $\delta$ denotes the analysis increment.
(2) Deduce pressure increment by use of the relationship between geopotential height increment and pressure increment:
\[
\delta p = \frac{p_1 - p_0}{\bar{T}(T_v)} \delta \phi_v,
\]
where \(T_v\) and \(R_u\) are virtual temperature and dry gas constant, respectively. To differentiate variables in Eq. (16) from balanced variables, the subscript “f” is used to denote basic variables, i.e., the first guess or background fields (the same below).

(3) Utilize the relation between pressure and nondimensional pressure
\[
\pi = \left(\frac{p}{p_0}\right)R_u/C_p,
\]
and consider effects of virtual temperature, the increment of balance part of nondimensional pressure \(\pi\) derived from pressure increment is
\[
\delta \pi_a = -\frac{\pi_f \delta \phi_a}{C_p(T_v)} - \delta \phi_a,
\]
and potential function \(\psi\) through Eq. (1).

We define state vectors of GRAPES model \(\mathbf{x}_m = (\psi, \mathbf{\Pi}, \mathbf{q})^T\), and analysis variables \(\mathbf{x} = (\psi, \mathbf{\Pi}_a, \mathbf{q}_a)^T\), where \(\psi, \mathbf{\Pi}, \mathbf{q}, \mathbf{\Pi}_a\), and \(\mathbf{q}_a\) are all vectors. The subscripts “m” and “a” denote model variables and matrix transposition, respectively. Thus, the relations between the model and analysis variables in Eqs. (2)-(4) and Eq. (7) can be established through transformation operator \(K_m\) as follows:
\[
\mathbf{x}_m = K_m \cdot \mathbf{x}.
\]

The transform in Eq. (20) is called balance transform, and \(K_m\) is balance operator. Ignoring the correlation of velocity potential and stream function, and transforming model wind components \(u, v\) into analysis stream function \(\psi\) and potential function \(\chi\), the balance transform matrix \(K_m\) at model levels can be expressed as
\[
K_m = \begin{pmatrix}
-\partial/\partial y & \partial/\partial x & 0 & 0 \\
\partial/\partial x & -\partial/\partial y & 0 & 0 \\
S_p M P_{\chi} & 0 & I & 0 \\
0 & 0 & 0 & I
\end{pmatrix}.
\]

For the potential temperature \(\theta\),
\[
\theta_t = P(\psi) = -\frac{g}{C_p} \frac{\partial \pi_t}{\partial z}.
\]

Currently, nonhydrostatic forecast models are usually integrated from hydrostatic balance initial state with vertical velocity \(w = 0\). In practice, as the variables \(\pi\) and \(\theta\) are not completely independent, the data assimilation scheme for the nonhydrostatic model only contains \(\pi\), while \(\theta\) is usually deduced from \(\pi\), and \(\tilde{w}\) is obtained by \(u\) and \(v\) through Eq. (1).

4. Observation operators

Observation operators set the physical constraints between analysis variables and observation data (Lorenc, 1986; Pailleux, 1990). Model variables \(\mathbf{x}_m\) are usually defined on model levels while observational variables are defined on isobaric surface levels, so some observational elements are not exactly the same as model variables. Thus, all kinds of observation operators in the m3DVAR system must be redesigned. Write the observation operator in a general form:
\[
H = H_p H_s,
\]
where \(H\) is observational operator; \(H_s\) is spatial interpolation operator, used in transforming model (control variables) at model grids into quantities at observation stations, including spatial transformation in both horizontal and vertical directions; \(H_p\) is physical transform operator, i.e., physical transform from model variables (control variables) to observational elements. Observation operator \(H\) in Eq. (24) contains a series of operators from state variables of model to equal observational variables in observational space. The following discussions are mainly focused on configuration
of observation operators in the m3DVAR scheme for nonhydrostatic models.

4.1 Spatial transform operator $H_s$

Spatial transform is conducted in horizontal and vertical directions. Horizontal transform from model variables to observational ones adopts the bilinear interpolation method, while vertical transform takes the cubic spline interpolation scheme to ensure the calculation precision. Observations are usually defined on isobaric surface levels, of which pressure value can be directly used as interpolation nodes of vertical transform, while pressure value of the model can be considered as cubic spline interpolation nodes for wind and humidity fields. Alternatively, it works as well to directly transform observational elements to variables in the altitude coordinate and then make the interpolation. The above two choices can be experimented separately for comparison. For the mass field, as mentioned above, the pressure is regarded to be observational variables, and the height variable to be the coordinate reference for observation positioning in the vertical direction. Thus, geopotential height among the observational variables is the interpolation nodes in the vertical transform, and pressure is the interpolation function. Pressure is transformed from model space to observational space based on the height value.

4.2 Physical transform operator $H_p$

In m3DVAR, the mass field of the forecast model is nondimensional pressure $\pi$, while observational elements in routine observations are pressure or temperature. In other words, they are not consistent with each other. Thus, physical transform of observation operator (including tangent linear and adjoint) is usually needed to transform inconsistent observations into the same physical variables. If observational element is pressure, the physical transform in observation operator is to put nondimensional pressure to pressure, i.e.,

$$\pi = (p/p_0)^{R/C_p}, \quad (24)$$

while the physical transform in tangent linear observation operator is

$$\delta \pi = \pi \frac{R}{C_p \rho} \frac{\delta p}{\rho}, \quad (25)$$

If observational element is temperature, the physical transform in observation operator firstly needs conversion from nondimensional pressure to temperature in term of

$$\frac{\partial \pi}{\partial z} = - \frac{g}{C_p \rho \theta}, \quad (26)$$

$$T = \pi \theta. \quad (27)$$

Its physical transform in tangent linear observation operator is

$$\delta \theta = \frac{C_p(\theta v)_L^2}{g} \frac{\partial \delta \pi}{\partial z}, \quad (28)$$

$$\delta T = \theta \delta \pi + \pi \delta \theta. \quad (29)$$

Taking physical transform like above, state vectors in model space can be transformed to the ones consistent with observational variables, so further solving of innovation and minimization of cost function can be executed.

Through new observation operators, the capability of assimilating observations of m3DVAR is completely consistent with that of p3DVAR. It means that the routine observations and satellite radiance data can be directly assimilated (Table 1). Meanwhile, the current version adds assimilation module of Bogus data to effectively improve the analysis quality of typhoon intensity and position, and can directly assimilate the elements produced by Bogus such as tangent wind, temperature, humidity, etc. The observation operator of satellite radiance still adopts RTTOV (Radiative Transfer for Television and Infrared Observation Satellite Operational Vertical Sounder) radiance transfer model, and the physical variables are also defined in model coordinate levels. It is obvious that observation errors for m3DVAR can stay consistent with p3DVAR. The quality control scheme is the same.

<table>
<thead>
<tr>
<th>Observations</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEMP</td>
<td>$u, v, T, H, RH$</td>
</tr>
<tr>
<td>SYNOP</td>
<td>$u, v, T, H$</td>
</tr>
<tr>
<td>SHIPS</td>
<td>$u, v, T$</td>
</tr>
<tr>
<td>AIREP</td>
<td>$u, v, T$</td>
</tr>
<tr>
<td>SATOB</td>
<td>$u, v$</td>
</tr>
<tr>
<td>TOVS</td>
<td>radiance</td>
</tr>
</tbody>
</table>
as that of p3DVAR, and these observations are considered to be invalid and then eliminated while the absolute value of difference between observation and background field is greater than background error at a certain extent.

5. Background error covariance

Background error covariance is a weighting function that allows cost function to adjust the effects of background fields. In variational data assimilation systems, the background error covariance usually corresponds to the statistics value of forecast error for a specific model, and it has to go through optimized adjustment in practice applications (Ingleby, 2001; Wu et al., 2002). If the background error covariance is realistic, the analysis results are the maximum likelihood estimation of the real atmosphere state. Obviously, it is very important to precisely obtain the background error covariance in the 3DVAR assimilation system (Daley, 1991). Since analysis variables of m3DVAR and p3DVAR are not consistent, the reconstruction of the background error structure is necessary for the new analysis system. There are two approaches to construct background error covariance matrix: (1) estimate the homogeneous and isotropic error structure of the variables on isobaric surface, then convert them to corresponding ones at model levels by coordinate transformation; and (2) adopt the NMC method (Parrish and Derber, 1992), i.e., obtain error structure of variables at model levels by directly using statistics model variables. Its advantages are easy to achieve and can provide the multiple correlation in the whole model areas. These two methods in statistical significance are the same, but the latter is more precise than the former in describing the structure character of forecast errors. Currently, the background error of m3DVAR system is obtained from the first method. Suppose the background error covariance can be separated into the horizontal and vertical parts, for the horizontal parts, we can calculate the horizontal correlation scale and utilize a recursive filter to the horizontally propagate background error; while for the vertical direction, we can obtain RMS (root-mean-square) of vertical mode and normalized eigenvector by EOF (empirical orthogonal function) decomposition method based on transformed model levels errors, and then jointly compose them to obtain the background error covariance matrix. The background error structure based on the latter method will be further investigated. In addition, flow dependent background errors based on ensemble forecast perturbations (Fisher, 2003) has achieved better results with numerical forecast models such as WRF (Skamarock et al., 2005). These studies have been paid much attention recently, which is also one of our further research interests.

6. Experiment results

To verify the performance of the m3DVAR system, we designed firstly idealized tests on single observation, carried out experiment of real observation data by m3DVAR and corresponding forecast model, and then compared the analysis results with that of p3DVAR. The experimental domain is 5°–65°N, 55°–145°E, while horizontal mesh sizes are 0.3°×0.3° with 31 vertical levels in these experiments. This section gives the analysis increments of the idealized test of single observation and the analysis results of real observation in GRAPES m3DVAR system.

6.1 An idealized test of a single observation

In 3DVAR data assimilation system, single variable analysis can exactly reflect the response of observations to background error and the impacts of background error and observation error on analysis, while multivariable analysis can investigate the physical constraints among different variables and the characters of analysis increment in the distribution and structure. In this section, the correctness of m3DVAR is verified through single element ideal test on single observation, and the property of background error covariance is further investigated. Based on the theory formula of 3DVAR (Courtier et al., 1998), for single observation ideal test on isobaric surface levels, analysis increment should propagate in accordance with the mode of Guass distribution along isobaric surface levels, and meet wind and pressure constraints, i.e., the analysis increment of wind and pressure approximately
satisfies dynamical balance relation. As for m3DVAR system, model coordinate and isobaric surface coordinate should be parallel when not considering terrain. Hence, the distribution of analysis increment of single observation should be consistent with that of isobaric surface.

In our single observation idealized test, we put one observation on the 15th σ-level without terrain at 30°N, 100°E at 5383.13-m altitude. We mainly study the propagating distribution mode of analysis increment along model levels and constraint relationship of wind and pressure fields. When observation is pressure $p$ and analysis variable is nondimensional pressure $\pi$, the distribution of analysis increment is circled in the center of observation with gradually decreasing towards side, and center value is consistent with analytic solution (first row in Fig. 3). Through dynamical balance relation, zonal wind increment $\delta u$ presents a distribution of zonal symmetry with an opposite wind direction, while meridional wind increment $\delta v$ also shows symmetric distribution structure with an opposite wind direction. Obviously, these results are consistent completely with that of theoretical analysis. The second and third rows in Fig. 3 show the corresponding analysis results, in which observation variables are zonal wind $u$ and meridional wind $v$ with analysis variable $\pi$. As shown in Fig. 3, distribution structures and values of analysis increment $\delta \pi$, $\delta u$, and $\delta v$ are fairly coherent with theoretical model when observation variable is wind components $u$ and $v$, respectively. Thus, the single observation test shows that the analysis results of m3DVAR can exactly and reasonably reflect the interactions among different variables.

6.2 Multi-variable assimilation of real observations

To analyze difference of model initial value

![Fig. 3. Analysis increments derived from single observation at (30°N, 100°E) by m3DVAR. The first, second, and third row observational variables are pressure $p$, $u$, and $v$; the first, second, and third column analysis variables are $\pi$, $u$, and $v$, respectively.](image-url)
obtained from m3DVAR and p3DVAR systems, we separately assimilate the TEMP data at 0000 UTC 14 October 2006 by the two systems and compare model initial values from the two analysis fields. The background fields of m3DVAR are the 12-h forecast fields of GRAPES model, and that of p3DVAR are the same time forecast fields in isobaric surface levels. Meanwhile, the valid TEMP data are completely consistent after the quality control for both assimilation systems. The analysis fields obtained from p3DVAR are transformed by vertical coordinate, horizontal staggering, and variable transformation through standard initialization module for GRAPES model (Huang et al., 2005) and obtain model initial value. Apparently, the analysis fields for the m3DVAR can be directly used as the model initial value. Figure 4 shows the difference of nondimensional pressure $\pi$ (Fig. 4a), $u$ (Fig. 4b), and $v$ (Fig. 4c) between the two model initial fields derived from m3DVAR and p3DVAR on the 15th $\sigma$-level. As presented in Fig. 4a, the difference of nondimensional pressure $\pi$ tends to be small at the lower terrain and ocean areas, while much larger at 35°N northward, 105°E westward, especially obvious at the plateau area with higher terrain. These features are pretty apparent for wind components $u$ and $v$ (Figs. 4a and 4b). The background fields of m3DVAR can be obtained from forecast model. Its analysis fields can be used as model initial value because of the same variable settings for m3DVAR and model; while in p3DVAR system, one must transform model variables from model coordinate to isobaric surface coordinate to obtain the background fields. After assimilation, the analysis fields are transformed from isobaric surface into $\sigma$ levels and eventually the initial value of forecast model is obtained. Furthermore, since there exist inconsistent variables between forecast and analysis fields, the physical transformation must be undertaken for p3DVAR. Obviously, vertical interpolation and variable transformation will increase

![Figure 4](image.png)

**Fig. 4.** The difference between the model initial fields derived from m3DVAR and p3DVAR with TEMP data on the 15th $\sigma$ A level at 0000 UTC 14 August 2006. (a) $\pi \times 10^{-4}$, (b) $u$ (m s$^{-1}$), and (c) $v$ (m s$^{-1}$).
errors in the model initial value as well as the imbalance among different variables such as wind and pressure, which will induce “spin up” of the model and reduce the forecast skill. Over plain areas, model initial values of the two systems have a relatively smaller difference caused by interpolation errors. The difference is approximately equal to initial errors caused by interpolating analysis fields from isobaric levels to model levels. In practice, the wind and pressure balance of m3DVAR is much more reasonable than that of p3DVAR.

We make a further investigation about innovation and analysis residual on single observation station to compare directly the difference of the two analysis systems. The innovation is the difference between observation and background fields, and analysis residual is the difference between analysis increment and observation. One sounding station is chosen in the plateau area where the difference is relatively larger (Fig. 4a), and the reasons for the larger difference are studied. Figure 5 shows the vertical profiles of innovation and analysis residual at Station 56046 which is located at 33.75°N, 99.65°E with a 3969-m altitude. In Fig. 5a, it can be seen that the innovation for $\pi$ on standard isobaric levels are basically identical between m3DVAR and p3DVAR, but obviously different on the significant levels, such as 425, 325, and 120 hPa. The innovation from p3DVAR exhibits unreasonable ladder-type vertical profiles (Fig. 5a). Because the vertical levels of observation are not consistent with isobaric levels of the p3DVAR system, and also, vertical levels are fewer than model levels, the values of model variables in the observation space have larger errors. Besides, vertical interpolation and physical transform of background fields may also lead to inaccurate innovation on significant levels. All these can directly affect the quality of analysis. For the m3DVAR system, the vertical profile of innovation is more acceptable (Fig. 5a). This is the main reason why the interpolation and physical transform are not included in m3DVAR, and why the error derivation is reduced. From Fig. 5b, it can be seen that there is a greater analysis residual caused by unreasonable innovation in p3DVAR than that of m3DVAR, especially on significant levels. The analysis residual can reflect how close the analysis value is to the observation. Figure 5b further shows that m3DVAR can obtain more reasonable analysis.

To verify the analysis quality of the two data assimilation systems, a group of short-range forecasts is conducted using the two different analysis fields as model initial value. A 30-day assimilation forecast is
carried out with observations in October 2006. Monthly averaged $T_s$ scores of 24-h precipitation forecasts (Fig. 6) show that the scores by m3DVAR are higher than that by p3DVAR for all precipitation types except for drizzle. This further exhibits that the m3DVAR system can provide better-quality model initial value than the p3DVAR system.

7. Summary and conclusions

In this study, the GRAPES m3DVAR is developed specifically for the nonhydrostatic model GRAPES based on the p3DVAR. The interpolation error in the transformation of background and analysis fields is effectively avoided, and the model initial value is improved. Meanwhile, the new system also provides a foundation for the further development of a 4DVAR assimilation system of GRAPES.

The analytical variables of the m3DVAR and the p3DVAR are not exactly the same, hence we redesign the analysis of the m3DVAR system. Dynamical balance constraints between mass and wind fields are reformulated, and an effective mathematical solution is proposed to avoid complicated problems caused by the nonlinear equations under the terrain-following coordinate. These measures can also effectively get over difficulties induced by solving the tangent linear equation and the adjoint equation and the nonlinear terms in the balance equation. The observation operator is reconstructed and observational elements of routine observations are consistent with model variables, which is more definite in theory and can reduce error derivation. The results from both the idealized tests with a single point observation and the real observation experiments show that treatment of the balance relation and the construction of observation operator are correct and reasonable.

In addition, the discretization scheme of horizontal Arakawa-C grids and Charney-Phillips vertical staggering grids in the m3DVAR are rearranged to match the nonhydrostatic GRAPES model, which avoids interpolation errors caused by transform of different grids. However, further experiments are still needed in order to debug and optimize assimilation effects of the m3DVAR before its application in operational numerical weather forecasts.

**Acknowledgments.** The authors would like to thank many colleagues from the Chinese Academy of Meteorological Sciences, especially Mr. ZHU Zongshen, for providing valuable help with this work.

**REFERENCES**


