ODD AND EVEN SYMMETRY OF ATMOSPHERIC CIRCULATION—PART II: MEASUREMENT AND ANOMALY ANALYSIS*

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ABSTRACT

First of all, for the odd and even symmetry components of both hemisphere circulations are given. The absolute and relative measurements with which to examine the seasonal change and interannual abnormality of the components on the 1980—1986 monthly mean 500-hPa height charts. Results show a noticeable relationship between a major El Nino event and the component anomaly, with the even part anomaly almost-synchronizing with its culmination and that of the odd part about half a year prior to the event, a relation that is still more sharply manifested at the 200-hPa height field.

Key words: atmospheric circulation, symmetry, measurement, anomaly analysis

I. INTRODUCTION

It is shown in Part I of this paper (Guan et al. 1994) that there exist considerable spacial / temporal odd and even symmetries (OESs) in the atmospheric environment and principal external forcing factors, which provide a reliable basis for OES analysis in a dynamic context. As judged from the odd / even components of the global monthly mean circulation symmetric about the equatorial plane, their climatic features can be clearly revealed in terms of the basic factors governing atmospheric circulations in a concise manner (Yeh and Zhu 1958; Pogoxin 1978).

Long-range weather prediction is concerned with the interannual anomaly of planetary—scale monthly average circulations. For this reason, the article presents a method for measuring OES components, followed by analysis of the abnormality of odd / even components of global monthly mean fields of some elements symmetric about the equatorial plane. Since all the global circulations under study are divided just into two parts for OES purpose, the degree of freedom is much smaller than that in the analysis of planetary circulations, e.g., harmonic and spheric function analysis, leading to high concentration of anomaly information. Therefore, the great anomaly in either kind of components can be assumed to be the substantial anomaly of circulations on a global basis.

II. MEASUREMENTS OF OES ANOMALY

Denote the monthly mean time series of a particular element as

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\[ F(\lambda, \varphi, t_m, t_v) \]
\[ \lambda = 0 \sim 2\pi, \quad \varphi = \varphi_s \sim \varphi_N, \quad t_m = 1 \sim 12, \quad t_v = 1 \sim n, \]
where \( \lambda \) is the longitude; \( \varphi \) the latitude in units of radians, with \( \varphi_s (\varphi_N) \) being the southern (northern) latitude and \( |\varphi_s| = \varphi_N \); \( t_m \) the ordinal number of a month; \( t_v \) the ordinal number of a year with \( n \) being the total number of years.

Normality is represented by the long-term average climatic state in the form
\[ \overline{F}(\lambda, \varphi, t_m) = \frac{1}{n} \sum_{i=1}^{n} F(\lambda, \varphi, t_m, t_v). \]

Abnormality or anomaly is defined as the departure from the normality in the form
\[ F'(\lambda, \varphi, t_m, t_v) = F(\lambda, \varphi, t_m, t_v) - \overline{F}(\lambda, \varphi, t_m), \]
where \( \overline{F}, F \) and \( F' \) can be decomposed, according to the scheme in Part I of this paper, into the odd and even components symmetric about the equatorial plane, designated by subscripts \( o \) and \( e \), respectively, so that
\[ \overline{F}(\lambda, \varphi, t_m) = \overline{F}_o(\lambda, \varphi, t_m) + \overline{F}_e(\lambda, \varphi, t_m), \]
\[ F'(\lambda, \varphi, t_m, t_v) = F_o'(\lambda, \varphi, t_m, t_v) + F_e'(\lambda, \varphi, t_m, t_v). \]

For any of the element fields \( G \) and \( H \) over the domains \( \lambda \in [0, 2\pi], \varphi \in [\varphi_s, \varphi_N] \), the internal product is defined as
\[ (G, H) = \int_{\lambda_s}^{\lambda_e} \int_{\varphi_s}^{\varphi_e} G(\lambda, \varphi)H(\lambda, \varphi)\cos\varphi d\lambda d\varphi. \]

And from the properties of odd and even functions, it is easy to prove that
\[ (\overline{F}_o, \overline{F}_e) = 0, \quad \text{for } t_m = 1 \sim 12; \]
\[ (F'_o, F'_e) = 0, \quad \text{for } t_m = 1 \sim 12 \text{ and } t_v = 1 \sim n, \]
which means that the OES components are orthogonal over the region.

In principle, the OES measurements can be established from Eq. (6). However, the regionally averaged \([\overline{F}_o], [F'_o] \) and \([F'_e] \) are involved in the even components \( \overline{F}_e, F_e \) and \( F'_e \) of Eq. (4), respectively, so that
\[ [G_e] = \frac{1}{2\pi(\sin\varphi_N - \sin\varphi_s)} \int_{\lambda_s}^{\lambda_e} \int_{\varphi_s}^{\varphi_e} G(\lambda, \varphi)\cos\varphi d\lambda d\varphi \]
is obtained from the algorithm for the \( G_e \) field over the region. Owing to the fact that \( G_e \) is homogeneous throughout the field \((\nabla_h [G_e] = 0)\), there exists vast difference between different elements or \( G_e \)'s of the same factor at various levels with the result that its existence is not just superfluous but obscures the essence of the problem, which leads us to separate \( G_e \) from the even component part of Eq. (4). Therefore, we have the following decompositions
\[ F_e = [\overline{F}_o] + [F'_e], \quad F'_e = [F'_o] + F''_e, \quad \text{and } F''_e = [F''_e] + F'''_e, \]
where the superscript * denotes the deviation of the field from the regional mean and because
\[ [G_0] = 0, \text{ we have} \]

\[ \bar{F}_o = F'_o^*, \quad F_o = F'_o^*, \quad F'_o = F'_o^*. \quad (9) \]

Through (8) and (9), we get

\[ \bar{F} = [\bar{F}] + \bar{F}'^*, \quad (10) \]

with \( \bar{F}'^* = \bar{F}_o^* + \bar{F}_e^* \) and \( t_m = 1 \sim 12; \)

\[ F = [F] + F'^*, \quad (11) \]

with \( F'^* = F'_o^* + F'_e^* \) and \( t_m = 1 \sim 12, t_v = 1 \sim n; \)

\[ F' = [F'] + F'^*, \quad (12) \]

with \( F'^* = F'_o^* + F'_e^* \).

Obviously, the OES components are orthogonal in the deviation fields of Eq. (10) through Eq. (12) such that

\[ (\bar{F}_o^*, \bar{F}_e^*) = 0, \quad (F'_o^*, F'_e^*) = 0, \quad (F'^*, F'^*) = 0. \quad (13) \]

Consequently, the following Parseval identities

\[ \| F'^* \|^2 = \| F'_o^* \|^2 + \| F'_e^* \|^2, \]

\[ \| F'^* \|^2 = \| F'_o^* \|^2 + \| F'_e^* \|^2, \quad (14) \]

hold, where \( \| \| \|^2 \) represents the squared mode with its algorithm exemplified by \( G \) in the form

\[ \| G \|^2 = \int_0^{2\pi} \int_0^{2\pi} G^2(\lambda, \phi) \cos \phi d\lambda d\phi. \quad (15) \]

Eq. (14) represents the basic relation for measuring the odd and even symmetric components. And for convenience, (1) we assume \( \| \bar{F}^* \|^2 \) to be the variance of long-term mean deviation (anomaly) field for the month \( t_m \), and \( \| \bar{F}_o^* \|^2 \) and \( \| \bar{F}_e^* \|^2 \) to be its odd and even components, respectively, which are denoted as \( S_o, S_{ao} \) and \( S_{ae} \), respectively for later use; (2) we make \( \| F'^* \|^2 \) represent the variance of the deviation for \( t_m \) of \( t_v \), and \( \| F'_o^* \|^2 \) and \( \| F'_e^* \|^2 \) the odd and even components, designated as \( S, S_o \) and \( S_e \), respectively; (3) we take \( \| F'^* \|^2 \) to be the variance of the deviation (anomaly) departure for \( t_m \) of \( t_v \), and \( \| F'_o^* \|^2 \) and \( \| F'_e^* \|^2 \) to be its odd and even components, denoted as \( S_d, S_{do} \) and \( S_{de} \), respectively. The technical term “deviation” (departure) refers to the difference between the element magnitude and its regional (long-term) mean. Thus, Eq. (14) can be rewritten as

\[ S_{ao} + S_{ae} = S_o, \quad S_o + S_e = S, \quad S_{do} + S_{de} = S_d. \quad (16) \]

By dividing both sides by the rhs (right hand side) quantity and denoting the ratio in terms of \( \rho \), we change Eq. (16) into the form

\[ \rho_{ao} + \rho_{ae} = 1, \quad \rho_o + \rho_e = 1, \quad \rho_{do} + \rho_{de} = 1. \quad (17) \]
Since the variance is always greater than or equal to zero, the value domain is [0, 1] for the lhs terms.

The parameters $\rho_{m}$ ($\rho_{o}$, $\rho_{e}$) and $\rho_{me}$ ($\rho_{mo}$, $\rho_{me}$) give the percentages of odd / even component variances in the related deviation - field variances of the long-term mean deviation (deviation and its departure, respectively) field variances for $t_{m}$ ($t_{o}$ of $t_{e}$), and the bigger the percentage, the more important the component.

It should be noted that, although the measurement of the OES information of numerical results was reported in previous studies (e.g., Yu et al. 1990), no analysis was given concerning the OES involved in actual climatic data.

Eqs. (16) and (17) provide an absolute ($S$) and a relative ($\rho$) measurement parameters, which will be employed in the following discussions.

In the present paper $S$ and $\rho$ are calculated from some elements in the context of 1980 - 1986 monthly mean grid point data of ECMWF for the sphere band (60° N, 60° S) and the box $\Delta \phi \Delta \lambda = 5^\circ \times 5^\circ$, with the aid of a trapezoidal integrating formula for approximate calculations.

For the comparison of analyses of the odd and even component anomalies, given is the climatic change in the measurement parameter of 500 hPa long - term mean height deviation (see Section III), which can be regarded as the supplement to the part I of this paper. Then the interannual anomaly of these elements and the causes are analysed (see Section IV).

II. SEASONAL VARIATION IN OES COMPONENTS

As an example, the 500 hPa multi-year mean deviation ($\overline{H}^*$) is considered. Figure 1 presents the seasonal change of its variance $S_{m}$ and odd (even) component variance $S_{mo}$ ($S_{me}$), of which the main features are as follows.

(1) The variance $S_{m}$ (Fig. 1a) is marked by a January maximum; an April - May minimum, roughly 25% smaller; an August - September weak peak and a September - October feeble valley, with a tiny difference between them. If the size of land / sea and the land characteristics are symmetric about the equatorial plane, $S_{m}$ would be a quasi - double periodic (with a half - year cycle) harmonic curve; if the earth - atmosphere system made an immediate response to solar radiation, then $S_{m}$ peaks should be at both solstices (22 December and 22 June) and equinoxes (23 March and 23 September). That the $S_{m}$ curve goes far away from the ideal state can be largely attributed to the land - sea size difference, the inhomogeneous distribution of land in both hemispheres and the land - sea difference in thermal properties (Guan et al. 1994).

(2) The odd / even component variances $S_{me}$ and $S_{mo}$ (Figs.1b and 1c), respectively, are characterized by $S_{me}$ bigger than $S_{mo}$ all the year round, which is attributed to the fact that lower - tropospheric comes chiefly from the underlying surface in the form of latent heat, sensible heat and long - wave radiation, and in other words, depends heavily on the thermal condition of the surface. Observations show that in both the Winter and Summer Hemispheres the tropospheric temperature gradient is directed polewards, thus leading to the fact that the even component variance of the 500 hPa height field is dominated in the deviation field. As shown by Yeh and Zhu (1958), on the other hand, the seasonal change of stratospheric temperature shows that its poleward gradient to be directed towards the poles (equator) in the winter (summer) half - year, which is responsible for the predominance of the odd variance in the deviation field for winter and summer months.
(3) The odd component variance $S_{oa}$ (Fig. 1b) shows a double-periodic oscillation with peaks (valleys) observed in mid-July and late January (late March to early April and late October to early November); the even variance $S_{oe}$ (Fig. 1c) displays a mono-periodic oscillation, with its maximum (minimum) occurring in late December to early January (in the first half of July). It should be noted that (i) for the Northern Hemisphere spring and summer (fall and winter) $S_{oa}$ has its minimum and maximum after vernal equinox and summer solstice (autumnal equinox and winter solstice), respectively, lagging by less than 20 days (over one month). This should be in connection with great difference in land / sea size and their latitudinal distribution between both hemispheres (land has its solar energy transmitted to the atmosphere more quickly); (ii) it seems that $S_{oe}$ should be in a double-periodic oscillation, with the maxima each around an equinox when solar incident radiation is completely symmetric about the equator (even-type symmetry) and the minima each occurring in the neighborhood of a solstice. However, the realistic $S_{oe}$ is found to be in a mono-periodic oscillation, a behavior that is far away from the expected condition. The cause lies in the uniform part of $H_{oe}$ in mid-latitude and given by

$$\langle H_{oe}^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} H_{oe}^2 \, d\lambda$$

(18)

that constitutes the main part of $S_{oe}$. It is known that $\langle H_{oe}^2 \rangle$ depends on poleward temperature gradient, which is $> 0$ throughout the year in the lower half of the troposphere, peaking in January. As a matter of fact, the even component indeed has a double-periodic oscillation, as given by the $\rho_{oe}$ curve (Fig. 2).
Fig. 2. Seasonal variation in the $S_{n}/S_{a}$ ratio in the 500 hPa monthly mean deviation field for 1980—1986.

IV. OES ANALYSIS OF INTERANNUAL ANOMALY OF ATMOSPHERIC CIRCULATION

As stated before, OES analysis deals with the interannual anomaly of planetary-scale circulations. From Eqs. (16) and (17) one can see that such abnormality can be displayed by the time series of either deviation $H^* (H_{\psi}^*, H_{\phi}^*)$ or departure deviation $H^* (H_{\psi}^*, H_{\phi}^*)$. In both cases the absolute measurement parameter (16) and the relative one (17) can be used to indicate the interannual anomaly of planetary circulations. The following is the interannual anomaly of monthly mean 500 hPa height treated using the departure deviation absolute measurement parameter.

(1) Analysis of 500 hPa height field. Figure 3 depicts the 500 hPa time-dependent $S_d$, $S_{do}$, and $S_{da}$, from which one can see that for early 1983 when the strongest El Nino event in this century (Pogoxin 1978) was at its crest, the $S_d$ curve shows a peak, indicating that the 500 hPa circulation displays significant absolute anomaly on a global basis, and the $S_{do}$ curve (Fig. 3c) peaks, too, suggesting that the abnormality is revealed largely by that of the even component. Of interest is $S_{do}$ (Fig. 3b) that is anomalously weak at the crest of the event but shows its peak around February 1982 prior to event. It would have a great meaning in the forecasting context had it a significant regularity.

(2) Analysis of 200-hPa height field. Figure 4 illustrates the time variations of 200-hPa $S_d$, $S_{do}$, and $S_{da}$, which present main characteristics, identical to, even more evident than those shown in Fig. 3.

V. SUMMARY

In the above we have discussed the measurement of OES components and their anomaly wherewith to analyze several characteristics of the time-varying (seasonal and interannual) variation of these components in the mid and upper tropospheres. Preliminary results show that the study of atmospheric circulation OES is of help to reveal the planetary circulation anomaly and establish the relation to the major abnormality of such external forcing factors as El Nino events. And it is quite possible that some amount of information on circulation anomaly can be separated therefrom for predictive use.
Fig. 3. Temporal curves for 500 hPa monthly mean height anomaly over 1980—1986, with the variances $S_A$, $S_B$, and $S_C$, shown in (a), (b) and (c), respectively (units: 10^{-9}$ gpm^2).

Fig. 4. As in Fig. 3, but for 200 hPa.

REFERENCES


