ATMOSPHERIC WAVE AND ITS PHYSICAL ESSENCE AS REVEALED BY CHANGED CORRELATION COEFFICIENTS AND CORRELATION FIELD

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ABSTRACT

Formal change is made of the correlation coefficient (COCOEF) expression, leading to a new formula consisting of Fourier spectral coefficients that indicates a direct connection between the new form and atmospheric dynamic equations, thus resulting in a dynamic equation represented by COCOEF, which is meaningful in exploring a large-scale dynamic process in terms of the correlation field because the connection revealed by the field can have dynamic explanation with the aid of the new formula.

Key words: correlation coefficient, atmospheric wave, dynamic equation

1. INTRODUCTION

It is a common practice to employ COCOEF between time series and reveal atmospheric wave characteristics by means of the correlation field (Wallace et al. 1981). Yet the coefficient magnitude and sign show only the outside relation between two series but fail to exhibit the intrinsic relationships between the physical quantities. What entrusts a new function to the COCOEF obtained connection is to offer physical interpretations between the quantities involved, which is particularly important in the study of synoptical meteorology and atmospheric circulation. The example can be that the physical explanations of some teleconnections given by Wallace et al. and Hoskins et al. 1981; Hoskins et al. 1983 have brought us to deeper understanding of atmospheric circulation changes.

In view of the importance of COCOEF in the circulation research, a problem arises: Can we combine the COCOEF with dynamic equations? An attempt is made to transform the COCOEF calculations with the dynamic equation(s), thereby forming a statistical–dynamic combination, with the aim to exhibit the dynamic implication given by the correlation field as a basis for the field to display the atmospheric wave structure.

II. FORM OF VARIATION IN COCOEF

Assume elements $\xi(t)$ and $\eta(t)$ to follow the variation:

1. $\xi(t)$ and $\eta(t)$ are set to be continuous over the domain $[a, b]$ or satisfy Dirichlet conditions in such a way that $\int_a^b \xi(t) dt$ and $\int_a^b \eta(t) dt$ hold;

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(2) $\xi(t)$ and $\eta(t)$ are let to constitute a periodic function in the form

$$
\xi(t + \beta - \alpha) = \xi(t), \quad \eta(t + \beta - \alpha) = \eta(t),
$$

\[ (1) \]

here the concept of continuation can be used. With period $T = \beta - \alpha$ and circular frequency $\omega = 2\pi / T$ we have Fourier series of $\xi(t)$ and $\eta(t)$ at $T$, denoted as

$$
\xi(t) = \bar{\xi} + \sum_{n=1}^{\infty} (a_n \cos \omega t + b_n \sin \omega t) = \bar{\xi} + \sum_{n=1}^{\infty} (a_n, b_n),
$$

$$
\eta(t) = \bar{\eta} + \sum_{n=1}^{\infty} (a_n \cos \omega t + b_n \sin \omega t) = \bar{\eta} + \sum_{n=1}^{\infty} (a_n, b_n),
$$

\[ (2) \]

and also

$$
\xi' = \xi(t - \tau) = \sum_{n=1}^{\infty} (a_n \cos \omega (t - \tau) + b_n \sin \omega (t - \tau) = \sum_{n=1}^{\infty} (a_n, b_n \tau),
$$

\[ (3) \]

$$
h'(t, \tau) = \xi'(t - \tau)\eta'(t),
$$

\[ (4) \]

$$
a_n^2 + b_n^2 = a_n^2 + b_n^2,
$$

\[ (5) \]

where $\xi'$ and $\eta'$ represent the deviation of $\xi$ and $\eta$ from $\bar{\xi}$ and $\bar{\eta}$, respectively, with the mean being zero at $T$; $h'(t, \tau)$ is the function determined by $\xi'$ and $\eta'$ in which $\tau$ is the time lag, defined over $[-T, T]$. $\xi'$ and $\eta'$ have their samplings at $T$ as

$$
\xi(t_0 - \tau), \xi'(t_1 - \tau), \cdots, \xi'(t_i - \tau), \cdots, \xi'(t_N - \tau);\n$$

$$
\eta'(t_0), \eta'(t_1), \cdots, \eta'(t_i), \cdots, \eta'(t_N).
$$

With $t_N = t_0 + T$, similarly, we find

$$
h'(t_0, \tau), h'(t_1, \tau), \cdots, h'(t_i, \tau), \cdots, h'(t_N, \tau),
$$

so that

$$
\sum_{i=1}^{N} \xi'(t_i - \tau)\eta'(t_i) = \sum_{i=1}^{N} h'(t_i, \tau),
$$

with the sampling interval

$$
\delta = t_{i+1} - t_i = \frac{T}{N}.
$$

As the sampling interval gets infinitesimal or continuous, an infinite number of samples is needed, i.e., $\lim_{N \to \infty} \delta = 0$. Consequently, the COCOEF has the from

$$
r_\tau = \frac{\sum_{i=1}^{N} \xi'(t_i - \tau)\eta'(t_i)}{\sqrt{\sum_{i=1}^{N} (\xi'(t_i - \tau))^2} \sum_{i=1}^{N} (\eta'(t_i))^2} = \frac{\sum_{i=1}^{N} \delta h'(t_i - \tau)}{\sqrt{\sum_{i=1}^{N} (\delta \xi'(t_i - \tau))^2} \sum_{i=1}^{N} (\delta \eta'(t_i))^2}.
$$

\[ (7) \]

For $\tau = 0$, Eq. (7) is none other than the usual COCOEF expression with a limited number of samples. Since

$$
\lim_{N \to \infty} \sum_{i=1}^{N} (t_i, \tau) = \int_{t_0}^{t_0 + T} h'(t, \tau)dt = \int_{t_0}^{t_0 + T} \xi'(t - \tau)\eta'(t)dt,
$$
\[
\lim_{t \to 0} \sum_{i=1}^{N} \xi_i^2 (t - \tau) \delta = \int_{t_0}^{t_0 + T} \xi'(t - \tau) \xi(t - \tau) dt, 
\]

Eq. (7) is changed into

\[
r_r = \int_{t_0}^{t_0 + T} \xi'(t - \tau) n'(t) dt \big/ \left( \int_{t_0}^{t_0 + T} \xi'^2(t - \tau) dt \int_{t_0}^{t_0 + T} n'^2(t, \tau) dt \right)^{1/2},
\]

based on the properties of the orthogonal triangle function system and Eq. (5), substitution of Eqs. (2) and (3) into (8) yields

\[
r_r = \sum_{n=1}^{\infty} \left( a_n c_n + b_n d_n \right) / \left( \sum_{n=1}^{\infty} \left( a_n^2 + b_n^2 \right) \right)^{1/2},
\]

thus giving the integral form of COCOEF \( r_r \) and the expression of Fourier coefficients. The amplitude in relation to \( n \omega \) is written as

\[
A_{cn} = \left( a_n^2 + b_n^2 \right)^{1/2}, \quad A_{sn} = \left( c_n^2 + d_n^2 \right)^{1/2},
\]

where the coefficient takes the form

\[
a_n = A_{cn} \cos(\varphi_{cn}), \quad b_n = A_{cn} \sin(\varphi_{cn}), \quad \varphi_{cn} = \arctan \frac{a_n}{b_n},
\]

\[
a_{nt} = A_{cn} \cos(\varphi_{cn} + n \omega t), \quad b_n = A_{cn} \sin(\varphi_{cn} + n \omega t),
\]

\[
c_n = A_{sn} \cos(\varphi_{sn}), \quad d_n = A_{sn} \sin(\varphi_{sn}), \quad \varphi_{sn} = \arctan \frac{d_n}{c_n},
\]

with \( \varphi_{cn} \) as the initial phase angle of the \( n \)th harmonic spectral component of \( \xi' \). We can also have

\[
r_r = \sum_{n=1}^{\infty} A_{cn} A_{sn} \cos(n \omega t + \varphi_{cn} - \varphi_{sn}) \big/ \left( \sum_{n=1}^{\infty} A_{cn}^2 \sum_{n=1}^{\infty} A_{sn}^2 \right)^{1/2}
\]

which can be represented in vectorial form. Set

\[
\vec{\xi}_{cn} = a_n \hat{i} + b_n \hat{j}, \quad \vec{n}_{sn} = c_n \hat{i} + d_n \hat{j}
\]

and we find

\[
r_r = \sum_{n=1}^{\infty} \vec{\xi}_{cn} \cdot \vec{n}_{sn} \big/ \left( \sum_{n=1}^{\infty} \vec{\xi}_{cn} \cdot \vec{\xi}_{cn} \sum_{n=1}^{\infty} \vec{n}_{sn} \cdot \vec{n}_{sn} \right)^{1/2}.
\]

Obviously, (8), (9), (12) and (14) are equivalent to each other, differing from the correlation for sequences consisting of a limited number of discrete samples. Nevertheless, they do not change the implication typical of COCOEF's.

It should be sum of some of the components in (8) to (12), \( r_r \) actually corresponds to the COCOEF obtained from filtered datasets. Further, the new COCOEF expression is more easily connected to atmospheric dynamic equations as compared to the traditional form, which will be dealt with later.

III. REPRESENTATION OF SINGLE-POINT CORRELATION

A single-point correlation is defined as that of the temporal series of an element at a height \( \rho_0 \), latitude \( \varphi_0 \), and longitude \( \lambda_0 \), with the time sequences at a base point and other ones for the same element. By referring to the above formulae, we let the time series of an element differ at different points so that patterns formed through a single-point correlation will be different with
various points as the base one. However, this is merely an outside thing. The single-point correlation field serves to reveal the teleconnection pattern (Wallic et al. 1981), which can be interpreted on a theoretical basis.

We take the time series of a streamfunction field and the base point \((\lambda_0, \varphi_0, \rho_0)\) and denote

\[
\vec{\Psi}_0 = [ia_n \cos \omega t - b_n \sin \omega t] + [ia_n \cos \omega t + a_n \sin \omega t] (\lambda_0, \varphi_0, \rho_0) = ia_{0m} + jb_{0m},
\]

where \(\vec{\Psi}_0\) is referred to as the harmonic wave spectral vector. And we get the correlation field of streamfunction in the form

\[
r = \sum \vec{\Psi}_m \cdot \vec{\Psi}_n / \left( \sum |\vec{\Psi}_0|^2 \right)^{1/2},
\]

where

\[
|\vec{\Psi}_0| = |\vec{\Psi}_0| (\lambda_0, \varphi_0, \rho_0).
\]

At extratropics a quasi-geostrophic relation exists between streamfunction and potential function \(\Phi\) while at low, especially equatorial, latitudes the relation is set up with the aid of the equilibrium equation \(\nabla \times (f \nabla \psi + \nabla \times V_\psi) = \nabla^2 \Phi\) where \(V_\psi\) represents the horizontal nondivergent wind vector. As an approximation, (16) indicates quite well the correlation field between geopotential height sequences for mid to low latitudes, and with great accuracy for extratropics in particular. In deriving the correlation pattern, in reality, other types of temporal series, as of arctic ice index, when neccesary, are applied to procure the wave spectral vector pattern in lieu of \(\vec{\Psi}_0\). Denote

\[
S_{\nu} = \vec{\Psi}_{\nu} / \left( \sum |\vec{\Psi}_{\nu}|^2 \sum |\vec{\Psi}_{\nu}|^2 \right)^{1/2},
\]

\[
r = S_{\nu} \cdot \vec{\Psi}_\nu, \quad \mu = -\ln 2 \sqrt{\sum \vec{\Psi}_\nu \cdot \vec{\Psi}_\nu}.
\]

Thus, \(r\) of Eq. (16) can be rewritten as

\[
r = \sum r_{\nu}.
\]

For the derivatives of \(S_{\nu}\) with respect to \(\tau\) and the spatial variables \(\lambda, \varphi,\) and \(\rho,\) separately, the reader is referred to the Appendix. Thus, we can put \(r_\nu\) directly into the model to investigate the physics of atmospheric oscillation displayed by the correlation.

IV. LOW FREQUENCY AND TELECONNECTION

Part of the features of atmospheric motion can be described by means of the vorticity equation. The following coordinates are employed (Hoskins et al. 1981)

\[
x = a\lambda, \quad y = a\ln \left( \frac{1 + \sin \varphi}{\cos \varphi} \right),
\]

such that

\[
\frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} = \frac{1}{\cos \varphi} \frac{\partial}{\partial x}, \quad \frac{1}{a \varphi} \cos \varphi \frac{\partial}{\partial \lambda} = \frac{1}{\cos \varphi} \frac{\partial}{\partial y}.
\]
\[ \nabla^2 = \frac{1}{a^2 \cos^2 \varphi} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \varphi^2} \right) = \frac{1}{\cos^2 \varphi} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) = \frac{1}{\cos^2 \varphi} \nabla^2 \]

cos \varphi = \text{sech} \left( \frac{y}{a} \right), \quad \sin \varphi = \text{tanh} \left( \frac{y}{a} \right),

(23)

\[ J(A, B) = \frac{\partial A}{\partial x} \frac{\partial B}{\partial x} - \frac{\partial A}{\partial x} \frac{\partial B}{\partial x}, \quad \nabla^2 A = \nabla \cdot \nabla A. \]

(24)

Therewith the vorticity equation has the form

\[ \frac{\partial}{\partial t} \nabla^2 \psi + J(\psi, \nabla^2 \psi) \text{ch}^2 \left( \frac{y}{a} \right) \nabla^2 (\frac{\partial \psi}{\partial y} + \frac{\partial \bar{\psi}}{\partial y}) + \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \frac{\partial \bar{\psi}}{\partial x} \nabla^2 \psi \]

\[ + \left( \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \nabla^2 \psi + \frac{df}{dy} \right) \frac{\partial \psi}{\partial x} \nabla \bar{R}_* \]

(25)

in which \( D \) stands for the residue, containing the effects of friction, convergence, and divergence, and \( D = 0 \) when there is no friction or divergence horizontally, at which the absolute vorticity conservation equation is used. Also, in Eq. (25) \( \frac{df}{dy} = \frac{2i}{a} \text{sech} \left( \frac{y}{a} \right) \) represents the term of beta effect.

The expression of the spectral coefficient vector coming from the expansion of Eq. (25) obtained by Fourier evolution of \( \psi \) through Eqs. (2), (3) and (15), the expression as used in the study of real physical problems in complex space, has the form

\[ -nw_k \nabla^2 \bar{\psi}_n + \text{ch}^2 \left( \frac{y}{a} \right) \left[ J(\bar{\psi}, \nabla^2 \bar{\psi}_n) + J(\bar{\psi}_n, \nabla^2 \bar{\psi}) \right] + \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \frac{\partial \bar{\psi}}{\partial x} \nabla^2 \psi \]

\[ + \left( \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \nabla^2 \psi + \frac{df}{dy} \right) \frac{\partial \psi}{\partial x} \nabla \bar{R}_* \]

(26)

\[ \text{ch}^2 \left( \frac{y}{a} \right) J(\bar{\psi}, \nabla^2 \bar{\psi}) + \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \frac{\partial \bar{\psi}}{\partial x} \nabla^2 \psi + \frac{df}{dy} \frac{\partial \bar{\psi}}{\partial x} \bar{R}_* \]

(27)

where

\[ \bar{R} = D - \text{ch}^2 \left( \frac{y}{a} \right) \left[ \frac{\partial \bar{\psi}_n}{\partial x} \frac{\partial \bar{\psi}}{\partial \varphi} \nabla^2 \bar{\psi}_n - \frac{\partial \bar{\psi}_n}{\partial \varphi} \frac{\partial \bar{\psi}}{\partial x} \nabla^2 \bar{\psi}_n \right] + \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \frac{\partial \bar{\psi}_n}{\partial x} \nabla^2 \bar{\psi}_n \]

indicating the influence of disturbance on the stationary wave structure; \( \bar{R}_* = R_0 + \bar{R}_* \) includes nonlinear effects—interaction between the vector \( D \) obtained from the \( D \) expansion and disturbance at a variety of time scales, whose expression is not given here. Eqs. (26) and (27) describe, separately, the laws governing the perturbation and mean fields and shown in Appendix, we have

\[ \frac{1}{a} (S_{\mu} \cdot \nabla^2 \bar{\psi}_n) + \text{ch}^2 \left( \frac{y}{a} \right) \left[ J(\bar{\psi}_n, S_{\mu} \cdot \nabla^2 \bar{\psi}_n) + J(\bar{\psi}_n, \nabla^2 \bar{\psi}) \right] + \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \frac{\partial \bar{\psi}_n}{\partial x} \nabla^2 \bar{\psi}_n \]

\[ + \left[ \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \nabla^2 \bar{\psi} + \frac{df}{dy} \right] \frac{\partial \bar{\psi}}{\partial x} \nabla \bar{R}_n - \text{ch}^2 \left( \frac{y}{a} \right) \left[ J(\bar{\mu}, \nabla^2 \bar{\psi}) + S_{\mu} \cdot \nabla^2 \bar{\psi}_n J(\bar{\psi}_n, \mu) \right] \]

\[ - \left[ \frac{1}{a} \text{sh} \left( \frac{2y}{a} \right) \nabla^2 \bar{\psi} + \frac{df}{dy} \right] \frac{\partial \bar{\psi}}{\partial x} \nabla \bar{R}_n = S_{\mu} \cdot \bar{R}_n \]

(28)

\[ S_{\mu} \cdot \nabla^2 \bar{\psi}_n = \nabla^2 \bar{R}_n - 2 \nabla \bar{\mu} \cdot \nabla \bar{R}_n + \bar{R}_n (\nabla \bar{\mu} \cdot \nabla \mu - \nabla^2 \mu) \]

(29)

After the summation for Eqs. (28) and (29) we get an equation in which the condition of COCOEF \( r_* \) is satisfied. With the l h terms of \( \mu \) merging into those on the rhs, it follows that
i) \( r \) meets the need of Eq. (25) which bears resemblance to that for Rossby waves so that the \( r \)-related spatial structure is analogous to the Rossby wave-associated one, at which time the \( \mu \) change, nonlinearly and forcing effect \( R_a \) are ignored; ii) \( r \)-described wave propagation is considered with respect to the time lag \( \tau \), suggesting that the propagation, if revealed by the correlation, has to depend on the lag correlation pattern; iii) the distribution of the disturbance variance has influence on the space structure of the correlation; iv) if, based on Eqs. (28) and (29), the conditions of boundary values of the \( R_a \) and \( \mu \) field and \( r \) are given together with the \( r_{th} \) pattern at a time lag \( \tau \), then it should be likely to get the evolution and pattern of \( r \) as a function of \( \tau \) through numerical integration.

It should be noted that Eqs. (28) and (29) result from the combination of a dynamic equation with statistical relation, as one of the intermediate approaches to interpreting the statistical outcome in virtue of the theoretical findings or partly so. On the other hand, the correlation–shown wave characteristics can display some of the features of the wave of the flowfield in an indirect fashion and even directly under certain conditions. And what are the conditions? Set

(a) the vector \( |K_x| \) to be small enough to be neglected;

(b) \( \left| \frac{1}{r} \nabla r \right| \ll |\nabla \mu| \).

To a certain extent, the assumptions are reasonable. First, as usual, we apply a dynamic model that is linear, nondivergent and nonfrictional, suggesting that a linear model is used to approximately investigate the variation in \( r \). Secondly, because the magnitude of \( \mu \) does not alter the position of \( r \) zero–isoline, the ratio of the space change of \( \sqrt{\sum_n |\Psi_n|^2} \) to the magnitude of its own is smaller than \( \left| \frac{1}{r} \nabla r \right| \), implying that the under-study area average \( \sqrt{\sum_n |\Psi_n|^2} \) is close to unity, and the spatial change of \( r \) is of the same order as that of its own, we thus obtain

\[
\frac{\partial}{\partial \tau} \nabla^2 r + \chi^2 \left( \frac{k}{a} \right) \left[ \frac{1}{a^2} \nabla^2 \Psi - \frac{\partial \Psi}{\partial x} \frac{\partial \theta}{\partial y} \nabla^2 r \right] + \left[ \frac{\partial}{\partial x} \left( \frac{2 \Psi}{a} \right) \frac{1}{a^2} \nabla^2 \Psi + \frac{2 \Omega}{a} \text{sech}^2 \left( \frac{k}{a} \right) \frac{\partial \theta}{\partial y} \right] = 0. \tag{30}
\]

For low–frequency atmospheric waves, the correlation–revealed wave characteristics can be interpreted by the barotropic two–dimensional Rossby wave theory only when the medium changes slowly and basic flow is zonal (Wallace et al. 1981; Hoskins et al. 1981). Assume

\[
r = Be^{\sigma \theta}, \tag{31}
\]

where \( \sigma \) represents the phase function, satisfying \( \partial \theta / \partial x = k \), \( \partial \theta / \partial y = l \) and \( \beta \theta / \partial \tau = -\sigma \), with \( k \) (l) denoting wavenumber in the \( x \) (y) direction. Then we have the dispersion relation for slowly changing wave associated with Eq. (31) in virtue of (30), which is in the form

\[
\sigma = \tilde{u}_y k - \beta_M k / (k^2 + l^2), \tag{32}
\]

where

\[
\tilde{u}_y = \frac{\partial \Psi}{\partial y} \text{ch}^2 \left( \frac{2}{a} \right), \quad \beta_M = \frac{2 \Omega}{a} \text{sech}^2 \left( \frac{2}{a} \right) + \frac{1}{a} \text{sh} \left( \frac{2}{a} \right)^2 \frac{\partial \Psi}{\partial y} + \text{ch} \left( \frac{2}{a} \right)^2 \frac{\partial \Psi}{\partial y}. \]
Fig. 1. Time–varying evolutions of low–frequency geopotential height. (a) on day 3, contoured at 0.89 dam; (b) on day 8 at 1.4; (c) on day 13 at 1.6; (d) on day 18 at 1.5.

Fig. 2. Correlation pattern obtained from low–frequency height with the base point at 60°N, 80°E. (a) synchronous correlation contoured at 0.10 dam; (b) 5–day lag correlation at 0.082; (c) 10–day lag at 0.082; (d) 15 at 0.083. The dotted, broken and solid contours denote the zero, negative and positive values, respectively.

indicate basic flow and beta factor, respectively, in a spheric atmospheric. It deserves emphasis that the frequency $\sigma$ should not be continuous for the filtering by $\sum_a \ldots$ and the assumption of the limited length of dataset used as the basic period are available in the $r_\tau$ expression. $\sigma$ can be approximately regarded as changing continuously, wherewith the wave propagation speed and group velocity are derived from Eq. (32), a result obtained under linear assumption which possesses an identical form to that of the dispersion relation (Hoskins et al. 1981) found by a linearized model of Eq. (25).

When phase speed or group velocity (E–W oriented) is greater than zero, the behavior of the wave or its energy transport in the correlation field is related to the sign of $\tau$ because of $X - X_0 = \int_0^\tau C_\rho dt$, indicating the dependence of the dynamic variation in the phase of the correlation upon the direction the time lag coordinate orients. For the basic point (with $\tau > 0$) denotes the lead correlation and v.v. And for $X - X_0 \big|_{\tau > 0} > 0$, $C_\rho > 0$ follows.
From the foregoing analysis, the correlation pattern from the height time series and responsible for the teleconnection has dynamic implication, especially for extratropics, and reflects some of the features of low-frequency waves after it has been filtered by $\sum$. 

The low-frequency (30–60 days, corresponding to $\sum_{n=12}^{12} \sum_{n=6}^{6}$) and the related lag correlation, shown in Figs. 1 and 2, respectively, are prepared in the context of 1986 ECMWF 500-hPa height data.

One can see that the low-frequency geopotential perturbation is characterized by backward ultralongwaves at higher latitudes. And what happens to the correlation (Fig. 2)?

Evidently, Fig. 2 have the characteristics of ultralongwaves, with the correlation phase propagating eastward at higher latitudes, meaning that $\int_{-\infty}^{\infty} C_{r} dr$ is positive. And because the lag correlation $\tau < 0$, the phase feature $C_{r} < 0$, too, showing that waves are physically ultralongwaves moving eastward, the findings that agree with those displayed in Fig. 1 (the pattern subsequent to day 15 not shown).

It should be emphasized that, whether the physical explanations are good enough regarding the correlation structure and the lag correlation patterns depends on the rational treatment of the correlation field from actual measurements and on the dynamic model used as well. If $\{\psi_{0}\} = \{\psi_{r}\}$ is identical to the correlation between the filtered $\{\psi_{r}\}$ and the series $\{\psi_{0}\}$. As a result, $\{\psi_{0}\}$ is the temporal sequence of some meteorological element with which to reveal its relation to atmospheric wave or to exhibit features of the wave structure using the correlation pattern, Eqs. (28) and (29) are responsible for interpreting the structure in terms of linear Rossby wave theory. Yet, as $\mu \approx a$ constant and $Rn$ is dropped, it is likely to explain some characteristics resulting from the correlation expanded by use of the slowly–changing Rossby wave theory.

V. CONCLUDING REMARKS

We have shown in this paper the transformation of the COCOEF formula under the condition that the time series constitutes a basic period in such a way that the COCOEF's are expressed as the function of spectral coefficients in the spectral analysis of the sequence at this period. The reason that the Fourier integral form is not adopted lies in the limited length of the dataset employed. To make the correlation evident $2 \triangle t$ the perturbation components are removed. Here $\triangle t$ denotes the interval when sampling.

It is common knowledge that correlation, only when related to physical processes, has implication. For atmospheric motions, the intrinsic cause of inter–element correlation is the existence of atmospheric dynamic. In the atmospheric circulation research, the single–point correlation pattern can be explained directly by the wave structure existing in the dynamic equation(s) represented by COCOEF. The correlation pattern and change versus time lag are associated with the time mean field.

The combination of COCOEF with dynamic equation(s) is that of statistical and dynamic relations on a methodological basis. No doubt, this is of significance to the revelation of large-scale atmospheric motion in terms of the correlation patterns.
APPENDIX

The derivatives of $S_{nt}$ with respect to time and space are obtained as follows.

It is known from Eq. (15) that $|\boldsymbol{\psi}_{nt}|^2 = a_{nt}^2 + b_{nt}^2 = (a_n^2 + b_n^2) \left[ \frac{v_n}{a_n + b_n} \right]$, is not the function of space and time lag $\tau$, such that

$$\frac{\partial S_{nt}}{\partial \tau} = \frac{\partial \psi_{nt}}{\partial \tau} / \left[ \sum_n |\psi_{nt}|^2 \sum_n |\psi_n|^2 \right]^{-\frac{1}{2}}, \quad (A1)$$

where

$$\frac{\partial \psi_{nt}}{\partial \tau} = n_0 a_{nt} i - n_0 b_{nt} i = n_0 \mathbf{k} / (n_0 a_{nt} i + n_0 b_{nt} i),$$

and we thus have

$$\frac{\partial S_{nt}}{\partial \tau} = j n_0 k / S_{nt}. \quad (A2)$$

The derivative of $S_n$ with respect to the space variable is obtained in the following way:

$$\frac{\partial S_{nt}}{\partial \lambda} = \frac{\partial \psi_{nt}}{\partial \lambda} / \left[ \sum_n |\psi_{nt}|^2 \sum_n |\psi_n|^2 \right]^{-\frac{1}{2}} + \frac{\psi_{nt}}{\sum_n |\psi_{nt}|^2} \frac{\partial}{\partial \lambda} \left[ \sum_n |\psi_n|^2 \right]^{-\frac{1}{2}}$$

$$= -\frac{1}{2} \sqrt{\sum_n |\psi_{nt}|^2} \frac{\partial}{\partial \lambda} \left[ \sum_n |\psi_n|^2 \right]^{-\frac{1}{2}} - \frac{1}{2} \sqrt{\sum_n |\psi_{nt}|^2} \sum_n |\psi_n|^2 \frac{\partial}{\partial \lambda} \left[ \sum_n |\psi_n|^2 \right], \quad (A3)$$

Therein $\mu = -\ln \left[ \sum_n |\psi_{nt}|^2 \right]$ of Eq. (19) indicates the variance of flowfield oscillation. $\mu / \partial \lambda \approx 0$ means the slow or uniform change of variance distribution. Thus, (A3) takes the form

$$\frac{\partial S_{nt}}{\partial \lambda} = \frac{\partial \mu}{\partial \lambda} S_{nt}, \quad \frac{\partial^2 S_{nt}}{\partial \lambda^2} = \left( \frac{\partial^2 \mu}{\partial \lambda^2} \right) S_{nt}, \quad (A4)$$

where $\lambda$ can be replaced by the other space variables for the related derivatives.

REFERENCES

